Effect of prestress levels in prestressed CFRP laminates for strengthening prestressed concrete beams: A numerical parametric study

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Prestressed concrete beams may provide benefits that reinforced concrete beams cannot provide, such as greater load-carrying capacity with less reinforcement, improved serviceability, controlled manufacturing quality, and reduced construction time on-site.

Loss of prestress, however, may cause considerable reduction of such advantages. Possible reasons for prestress loss include corrosion and impact damage in the prestressing strands. In such cases, conventional repair methods may need extensive repair time and materials to recover the prestress-loss effects (for example, external prestressing cables anchored by concrete corbels installed on the tensile side of a damaged prestressed concrete beam).

Editor's quick points

- This paper presents the effect of various prestressing levels on the prestressed carbon-fiber-reinforced-polymer (CFRP) laminates for strengthening prestressed concrete beams.

- Calibrated nonlinear, three-dimensional finite-element-analysis models were used to predict the flexural behavior of the strengthened beams.
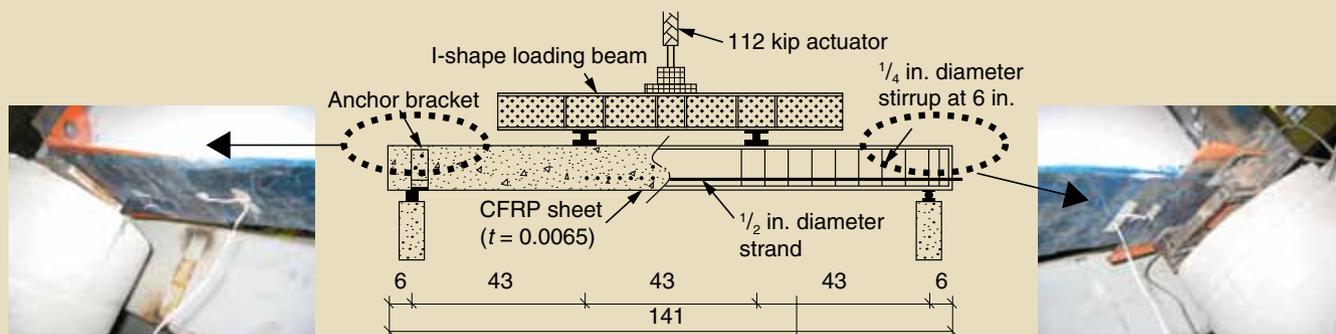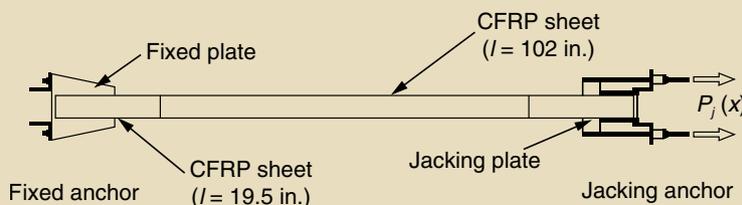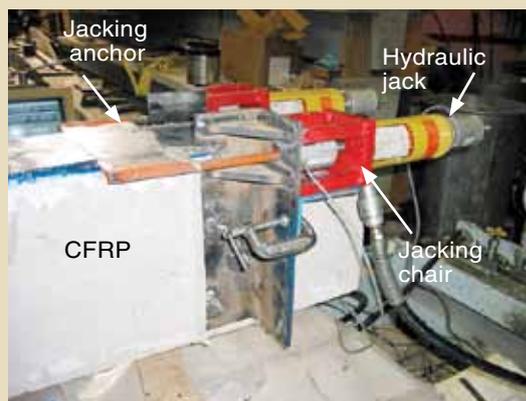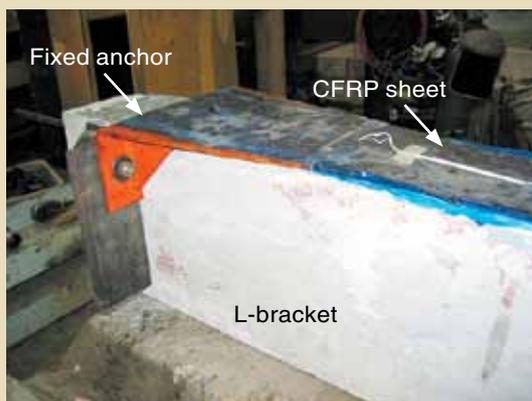
- A prestress level between 20% and 30% of the ultimate design strain of CFRP laminates is recommended for prestressed concrete beams strengthened with prestressed CFRP laminates.

Figure 1. These photos and diagrams illustrate the test setup and anchorage details (upside down for prestressing application). Note: CFRP = carbon-fiber-reinforced polymer; l = length of carbon-fiber-reinforced-polymer laminate; $P_j(x)$ = jacking force; t = thickness. 1 in. = 25.4 mm; 1 kip = 4.448 kN.

Recent advances in construction materials have contributed to the development of more-effective and more-efficient structural rehabilitation methods. Fiber-reinforced-polymer (FRP) composites, consisting of unidirectional fibers and matrix resins, are one example. FRP laminates have a high tensile strength-to-weight ratio, are noncorrosive, can be applied quickly on-site, require reduced long-term maintenance expenses, are resistant to fatigue, and can reduce labor costs.[1–5] Such repair materials can be simply bonded on the tensile sides of deteriorated concrete beams to compensate for the insufficient load-carrying capacity. FRP laminates may be prestressed to enhance the serviceability of concrete beams to which they are added or to recover the loss of prestress effects of structurally deficient prestressed concrete beams. Carbon FRP (CFRP) may be

an ideal candidate for prestressing applications, taking into account its excellent long-term performance.[1]

Typically, three methods are reported on prestressing FRP laminates:[6]

- **Camber method.**[7] The target concrete beam is artificially loaded against gravity using a hydraulic jack, and FRP laminates are bonded on the tensile soffit of the beam. After the FRP laminates are cured, the camber effect is removed so that prestress is naturally applied to the FRP laminates.

- **External reaction frame method.**[8] An external apparatus applies prestress to FRP laminates before bond-

Table 1. Reinforcement details

Beam	Reinforcement		Prestress level		Remarks
	Steel, in.^{2}	CFRP, in.^{2}	Steel, %*	CFRP, %	
B-1	0.23	n.a.	70	n.a.	E/F
B-2	0.15	n.a.	70	n.a.	E/F
B-3	0.15	0.038	70	50†	E/F
B-3-50	0.15	0.038	70	50‡	F
B-3-40	0.15	0.038	70	40	F
B-3-30	0.15	0.038	70	30	F
B-3-20	0.15	0.038	70	20	F
B-3-10	0.15	0.038	70	10	F
B-3-0	0.15	0.038	70	0	F

* Percentage of the ultimate strength (270 ksi) of prestressing strands
† Percentage of the actual failure strain (1.88%) of CFRP
‡ Percentage of the design failure strain (1.67%) of CFRP
Note: CFRP = carbon-fiber-reinforced polymer; E = experimental; F = finite-element analysis; n.a. = not applicable. 1 in. = 25.4 mm; 1 ksi = 6.895 MPa.

ing the laminates, then the prestressed FRP laminates are bonded on the target surface. After curing, the apparatus is removed.

- **Direct tensioning method.**[6,9,10] An anchor system is directly mounted on the target concrete beam, and prestress is applied against the beam itself. After the FRP system is cured, the anchorage can be left on-site[6] or can be removed, taking into account the aesthetics of the strengthened beam.[9,10]

Of these prestressing techniques, the direct tensioning method is recommended for practical application on-site.[11–13]

Despite the structural advantages of using prestressed CFRP laminates for strengthening concrete structures,[1,6,8] the contribution of prestress levels in the CFRP laminates has not been discussed in previous research. This paper reports the effect of various CFRP prestress levels on the flexural behavior of prestressed concrete beams externally strengthened with prestressed CFRP laminates. A three-dimensional (3-D), nonlinear finite-element-analysis (FEA) model was constructed to predict the flexure of the beams, including experimental validations.

Experimental program summary

The experimental program[14] consisted of three simply supported, medium-scale, prestressed concrete beams (6^1/$_4$ in. [160 mm] wide × 11 in. [280 mm] deep × 141 in. [3600 mm] long) subjected to a monotonic four-point bending

load. **Figure 1** and **Table 1** show detailed descriptions of the test beams.

The control beam, B-1, had two prestressing tendons (^{3}/$_8$ in. [9 mm] and ^{1}/$_2$ in. [13 mm]) for a total prestressing steel cross-sectional area A_p of 0.23 in.^{2} (148 mm^2). Beam B-2 included only one prestressing tendon (^{1}/$_2$ in. [13 mm] with A_p of 0.15 in.^{2} [97 mm^2]) to represent a significant loss of the prestressing effect.

Beam B-3 had the same internal reinforcing scheme as beam B-2 but was also strengthened with a single layer of prestressed CFRP laminate, which had a cross-sectional area A_f of 0.038 in.^{2} (25 mm^2) (**Fig**. 2). A single-ply, 19.5-in.-long (500 mm) CFRP laminate was additionally bonded on the anchor plate to avoid stress concentrations at the end of the plate (Fig. 1).

The prestressing strands were stressed to 70% of their ultimate strength (f_{pu} = 270 ksi [1860 MPa]). The external CFRP laminate was prestressed to about 50% of the ultimate design strain (ε_{fu} = 1.67%). **Table 2** shows detailed material properties.

To prestress the CFRP laminate, an integrated anchorage system was used (Fig. 1). The CFRP laminates were bonded to the fixed plates and jacking anchor plates using an epoxy adhesive, and the cured CFRP laminates were tensioned to the desired level of prestress (Table 1) against the L-brackets mounted on the target beam. The prestressed CFRP laminates were then bonded on the soffit of the beam and cured.
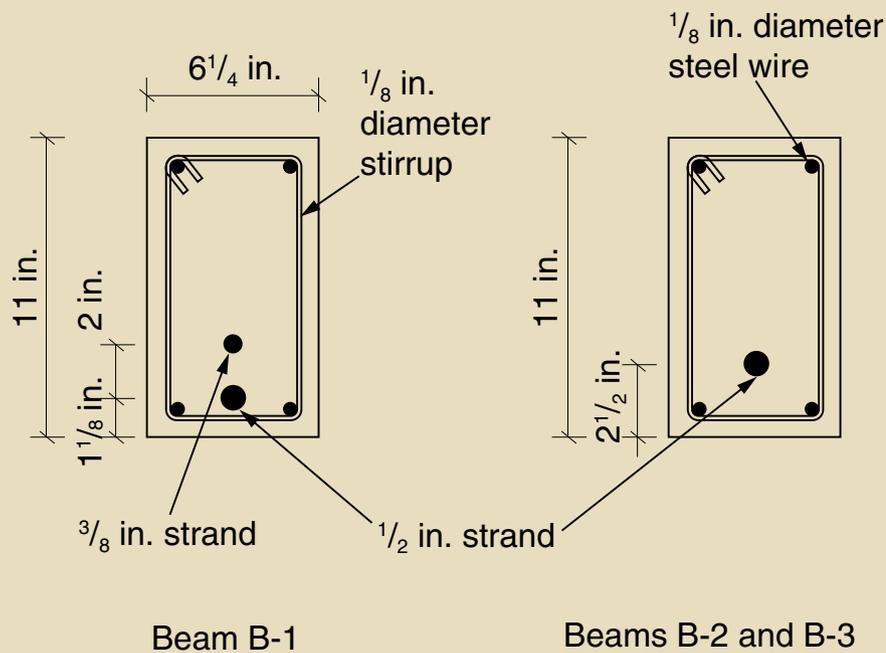
Figure 2. The beam details are shown in the cross-section diagrams. Note: 1 in. = 25.4 mm.

Table 2. Material properties

Material		Compressive strength, ksi	Tensile strength, ksi (specified strength)	Elastic modulus, ksi
Concrete[*]		5.4	0.52	n.a.
Steel strands[*]		n.a.	256 (270)	27,600
CFRP[†]	Actual	n.a.	620	33,000
	Design	n.a.	550	33,000
Epoxy[†]		12.5	7.9[‡]	440

Sources: Data from Shi 2003; MBrace, *MBrace Composite Strengthening System Design Guide* (Cleveland, OH: Master Builders Inc., 1998); MBrace, *MBrace Composite Strengthening System Engineering Design Guidelines* (Amherst, NY: Watson Bowman Acme Corp., 2002).

[*] Tested by Shi

[†] Manufactured by MBrace

[‡] Yield strength

Note: CFRP = carbon-fiber-reinforced polymer; n.a. = not applicable. 1 ksi = 6.895 MPa.

Kim et al.[15] discusses further details of the anchorage and beam strengthening. The strengthened beam B-3 successfully recovered the load-carrying capacity of the one-tendon beam, B-2, to the level of the undamaged two-tendon control beam, B-1. Failure of the strengthened beam, B-3, was due to progressive rupture of the CFRP laminate (**Fig. 3**). The permanent anchor system effectively precluded premature delamination failure of the CFRP laminates until the strengthened beam was loaded to failure in flexure.

Numerical modeling

Description of the model

A general-purpose commercial FEA program ANSYS was used to predict the flexural behavior of test beams and was used further for a parametric study. **Figure 4** shows a typical FEA model. The concrete was represented by eight-node composite elements with three translational
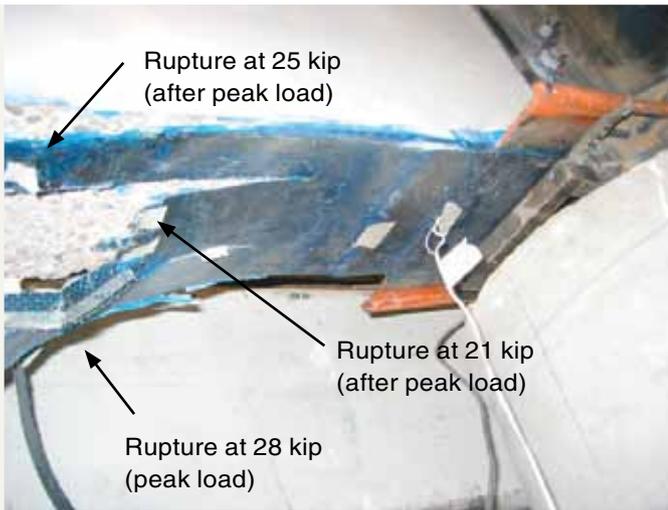
Figure 3. This photo shows the progressive rupture failure of carbon-fiber-reinforced-polymer laminates. Note: 1 kip = 4.448 kN.

Rupture at 25 kip
(after peak load)

Rupture at 21 kip
(after peak load)

Rupture at 28 kip
(peak load)

degrees of freedom per node. This element can simulate the crushing and cracking behavior of the concrete.

The William and Warnke approach was used to represent the behavior of concrete.[16] The prestressing strands were modeled with two-node spar elements with the same degrees of freedom as in the composite element. For simplicity, a bilinear stress-strain relationship was assumed for the prestressing strands (Fig. 4). The unidirectional CFRP laminate was modeled with two-node spar elements. Thus, the actual orthotropic material characteristics of the CFRP were simplified with uniaxial ones. The CFRP laminate

showed a linear stress-strain relationship until its rupture occurred (Fig. 4). The actual CFRP properties were used to predict the test beam B-3; however, the guaranteed design CFRP properties were used for the parametric study (Table 2) because these design properties would be used for actual design and practice. A perfect bond was assumed between materials. Premature delamination failure was not taken into consideration in the models because such a failure mode was not observed in the laboratory test due to the end anchorages.

CFRP laminate prestress levels

Various levels of prestress in the CFRP laminates (Table 1) were simulated by changing the initial strain of the two-node spar element. The initial strain is defined as Δ/L, where L is the element length (distance between two nodes) and Δ is the difference between the prestrained element length and the zero-strain element length. This initial strain is used in determining the stiffness matrix for the first cumulative iteration so that the prestressing effect is directly applied to the beam models. For convenience, the level of prestress in the CFRP laminate was quoted as a percentage of the ultimate design strain ($\varepsilon_{fu} = 1.67\%$). For example, B-3-30 indicated a beam with the same reinforcing scheme as beam B-3 and a prestress level of 30% in the CFRP laminate. A total of nine prestressed concrete beams were simulated, and six of them included the prestrain effect of CFRP laminates (Table 1).

FEA model

f

$(f_{pu}, \varepsilon_{pu})$

$(0.87f_{pu}, 0.28\varepsilon_{pu})$

ε

Steel strands

f

$(f_{fu}, \varepsilon_{fu})$

ε
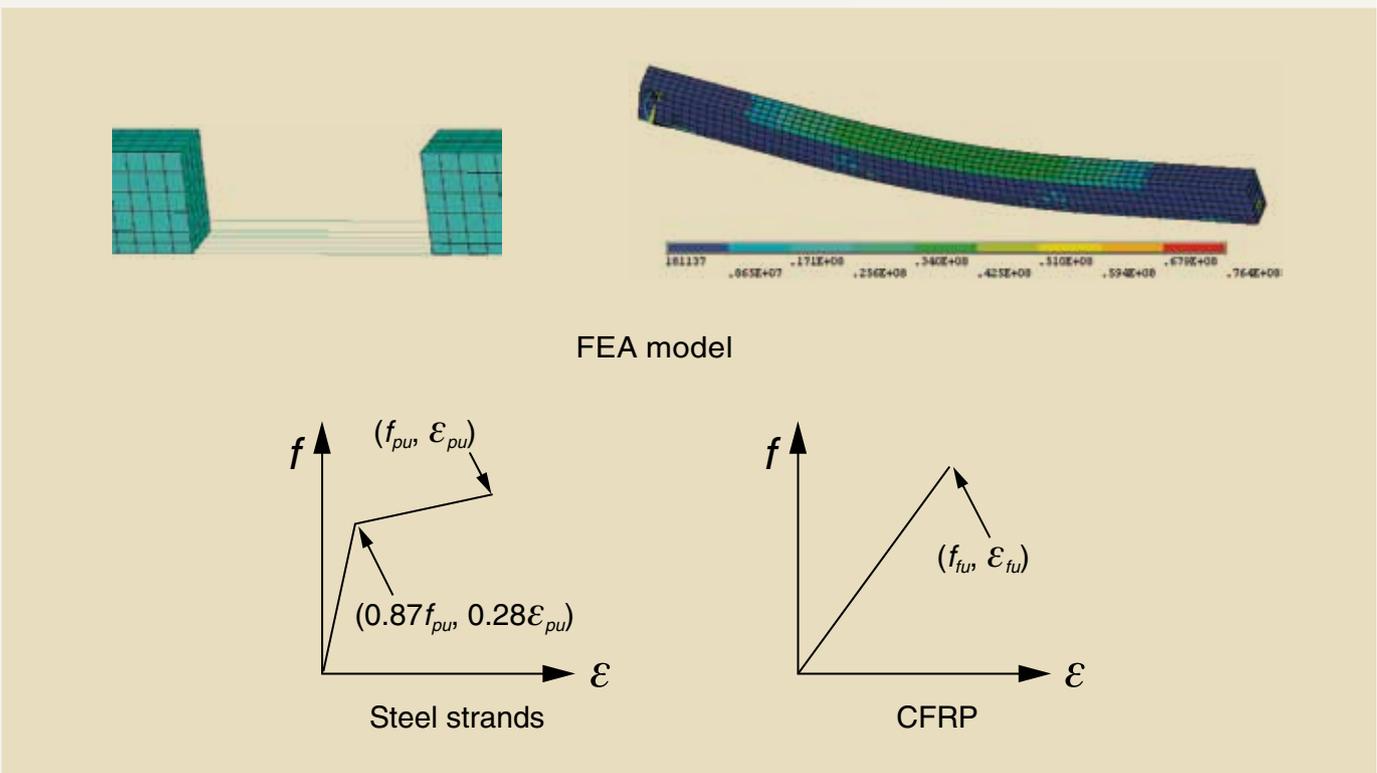
CFRP

Figure 4. These diagrams illustrate the finite-element-analysis (FEA) model and constitutive models for the reinforcement. Note: Measurements for the FEA model are in N/m². CFRP = carbon-fiber-reinforced polymer; f = stress; f_{fu} = ultimate strength of carbon-fiber-reinforced polymer; f_{pu} = ultimate strength of prestressing strands; ε = steel strain; ε_{fu} = ultimate failure strain of CFRP laminates; ε_{pu} = ultimate strain of prestressing steel. 1 N/m² = 145.038 psi.
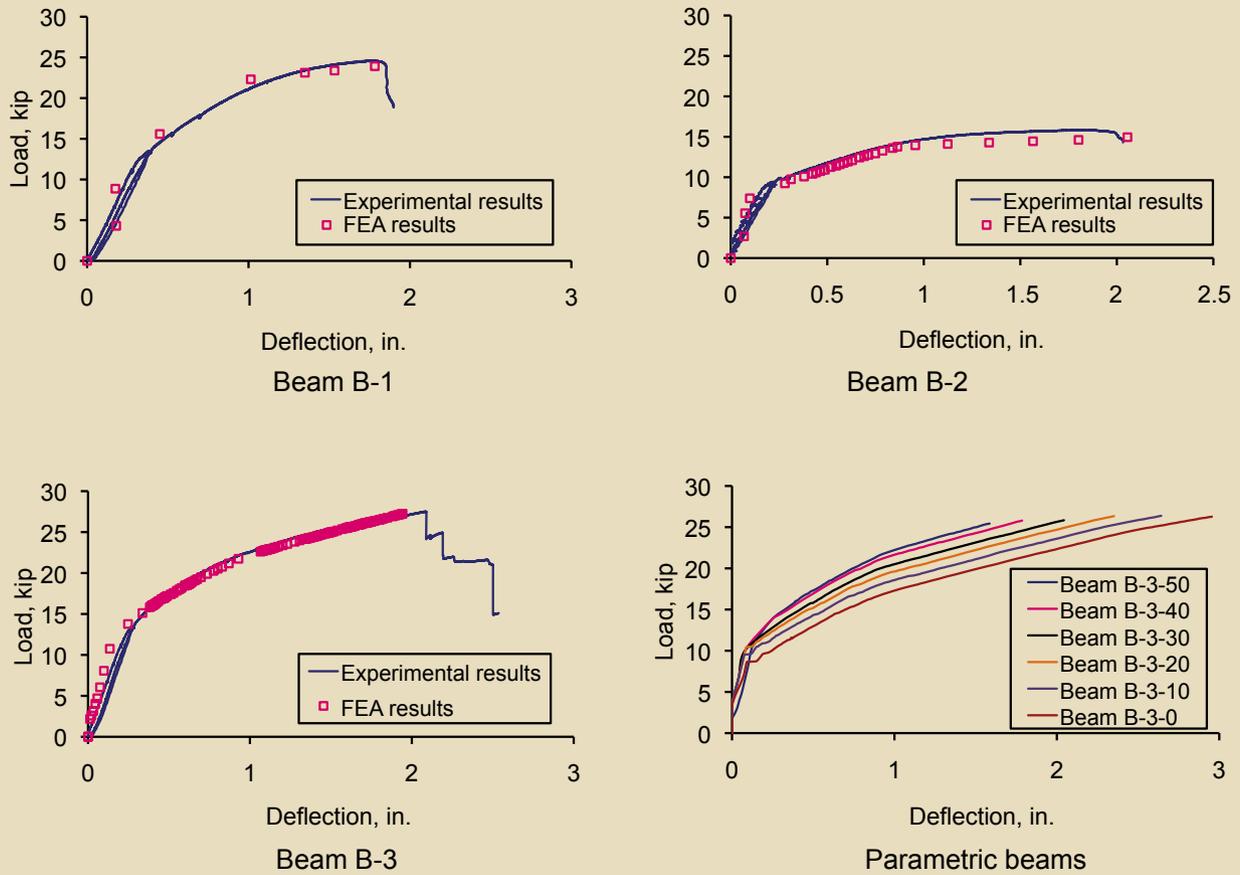
Figure 5. These graphs show the load-deflection responses at midspan. Note: FEA = finite-element analysis. 1 in. = 25.4 mm; 1 kip = 4.448 kN.

Table 3. Summary of flexural behavior

Beam	Cracking				Yielding				Ultimate			
	P_{cr}, kip		δ_{cr}, in.		P_{yld}, kip		δ_{yld}, in.		P_{ult}, kip		δ_{ult}, in.	
	EXP	FEA	EXP	FEA	EXP	FEA	EXP	FEA	EXP	FEA	EXP	FEA
B-1	12.8	13.7	0.28	0.37	22.4	22.4	1.16	1.01	24.6	24.0	1.73	1.78
B-2	7.6	7.4	0.12	0.10	14.1	13.9	0.87	0.96	15.9	15.0	2.03	2.06
B-3*	12.1	13.9	0.23	0.25	22.2	22.6	0.92	1.06	27.6	27.3	2.09	1.94
B-3-50†	n.a.	13.9	n.a.	0.25	n.a.	22.6	n.a.	1.06	n.a.	25.5	n.a.	1.59
B-3-40†	n.a.	13.4	n.a.	0.19	n.a.	21.5	n.a.	0.94	n.a.	25.8	n.a.	1.75
B-3-30†	n.a.	13.4	n.a.	0.30	n.a.	20.2	n.a.	0.93	n.a.	25.8	n.a.	2.04
B-3-20†	n.a.	10.5	n.a.	0.13	n.a.	19.0	n.a.	0.92	n.a.	26.4	n.a.	2.35
B-3-10†	n.a.	10.3	n.a.	0.14	n.a.	17.0	n.a.	0.79	n.a.	26.4	n.a.	2.64
B-3-0†	n.a.	9.2	n.a.	0.17	n.a.	15.9	n.a.	0.81	n.a.	26.2	n.a.	2.96

* Actual failure properties

† Guaranteed design properties provided by the manufacturer and used in finite-element analysis

Note: EXP = experimental; FEA = finite-element analysis; P_{cr} = cracking load; P_{ult} = ultimate load; P_{yld} = yield load; n.a. = not applicable; δ_{cr} = deflection at cracking load; δ_{ult} = deflection at ultimate load; δ_{yld} = deflection at yield load. 1 in. = 25.4 mm; 1 kip = 4.448 kN.
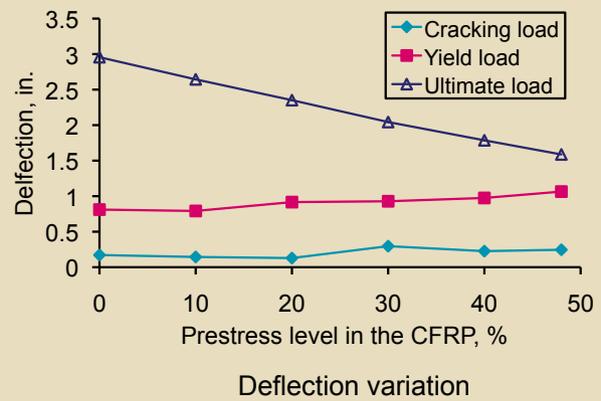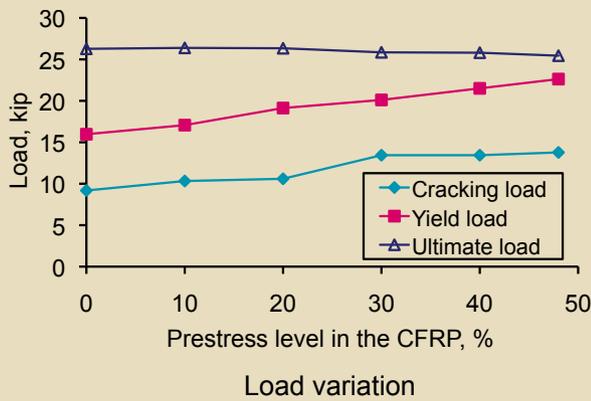
Figure 6. These graphs show the effect of carbon-fiber-reinforced polymer (CFRP) prestress levels on the variations of flexural responses. Note: 1 in. = 25.4 mm; 1 kip = 4.448 kN.
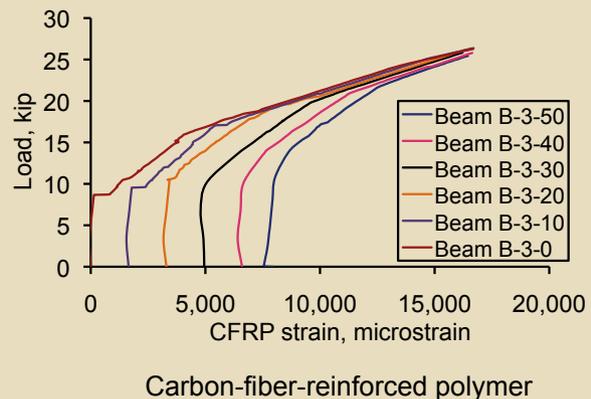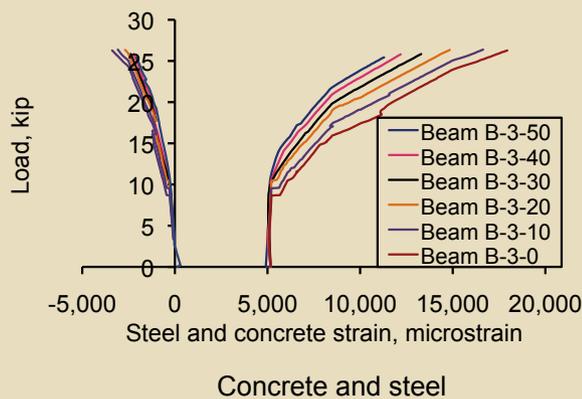

Figure 7. These graphs show the load-strain responses at midspan. Note: CFRP = carbon-fiber-reinforced polymer; microstrain = *x 10⁻⁶*. 1 kip = 4.448 kN.

Simulation analysis

Load-deflection response

Figure 5 shows a typical comparison of load-deflection responses between the FEA models and the test beams. **Table 3** also summarizes important flexural values. The FEA models provided good predictions against the experimental data, including average errors of 8.2%, 1.2%, and 2.8% for the cracking, yielding, and ultimate loads, respectively (Table 3 and Fig. 5). The flexural response of the test beams was measured from zero deflection. Thus, the initial upward camber deflections of the FEA results induced by the prestressing were shifted to zero to provide an appropriate comparison with the test data. Beam B-3 in Fig. 5 clearly shows the progressive rupture of the CFRP laminate (Fig. 3) in terms of the stepwise decrease in the load beyond its peak value of 27.6 kip (123 kN).

The prestress levels in the CFRP laminates significantly contributed to the load-deflection responses of the strengthened beams, as shown in the parametric beams (Fig. 5).

Figure 6 shows the variations of load and deflection characteristics of the strengthened beams, depending on the level of prestress in the CFRP laminates. A slight increase of cracking and yield loads was observed when the prestress level in the CFRP laminate increased; however, the ultimate-load-carrying capacity of the strengthened beams was not influenced by the prestress level. The effects of various prestress levels in the CFRP laminates on the deflections at cracking and yield loads were not remarkable, whereas those on the ultimate deflections were significant (Fig. 5 and 6). This phenomenon is due to the fact that the usable CFRP strains decreased when the initial prestress level increased. This was related to the bottom strain development of the beams, which directly affected the development of curvature.

Strains in the reinforcement

Figure 7 shows typical strain development in the internal and external reinforcement of the FEA beams. No significant strain changes were observed in the prestress-
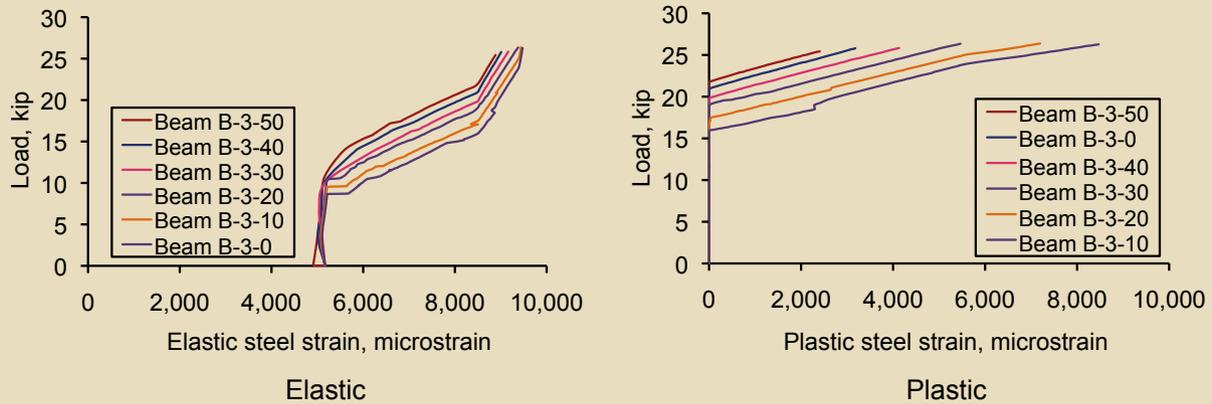
Figure 8. These graphs compare the strain increment rate of the prestressing strands. Note: microstrain = $x\ 10^{-6}$. 1 kip = 4.448 kN.
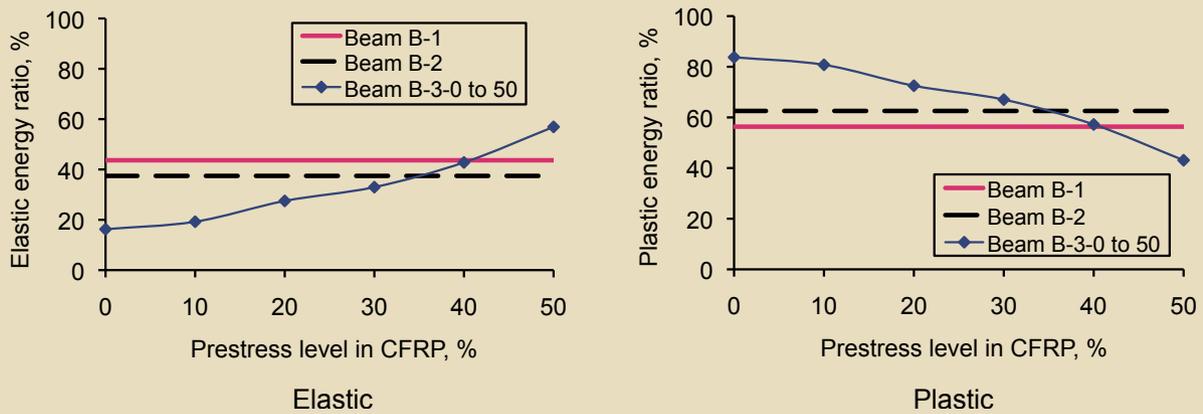
Figure 9. These graphs show the variation of energy ratios. Note: CFRP = carbon-fiber-reinforced polymer.

ing strands until the initial cracking load of the beams; however, the strain increment in the strands beyond the cracking load was considerably influenced by the level of prestress in the CFRP laminates. This is due to the stress-sharing mechanism between the internal and external reinforcement. The variation of CFRP strains was essentially dependent on the initial prestress levels in the laminates. As more load was applied, the CFRP strains converged to the same point with different strain increments (Fig. 7). This phenomenon explains the constant ultimate failure load of strengthened beams, which was explained in Fig. 6.

Figure 8 shows the development of elastic and plastic strains in the prestressing strands. The elastic strain increment rate significantly changed before and after yield loads of the strengthened beams. The rate of the plastic strain development in the prestressing strands was almost constant, irrespective of the level of prestress in the CFRP laminates; however, the strain values of the strands were significantly influenced by the CFRP prestress levels. This is attributed to the fact that the remaining usable CFRP strain capacities

after yielding of the beams were dependent on the initial prestress levels.

Variation of the energy

As discussed in the previous sections, the level of prestress in the CFRP laminates significantly influenced the flexural behavior of strengthened beams, including the elastic and plastic responses. For the present study, the total energy E_{total} and elastic energy $E_{elastic}$ were defined as the total area under the load-deflection curve and the area until yielding of the beam occurred, respectively. The plastic energy $E_{plastic}$ was then calculated by subtracting $E_{elastic}$ from E_{total}. The energy ratio was correspondingly defined as the ratio of the elastic or plastic energy to the total energy absorbed until the beam failed. **Figure 9** and **Table 4** show the variation of elastic and plastic energy ratios, including variations of the stiffness in the beams.

For the unstrengthened beams, 17% more elastic energy was observed in the two-tendon control beam, B-2, com-

Table 4. Summary of energy absorption and stiffness

| Beam | Energy, kip-in. | | | | | | Stiffness, kip/in. | | | | | |
| | Total | | Elastic | | Plastic | | Before crack | | Before yield | | After yield | |
	EXP	FEA	EXP	FEA	EXP	FEA	EXP	FEA	EXP	FEA	EXP	FEA
B-1	3.1	31.5	21.8	13.7	9.4	17.8	34.5	33.9	9.8	14.4	2.3	5.7
B-2	21.8	25.4	9.1	9.5	12.6	15.9	41.9	74.7	8.6	7.5	1.7	1.1
B-3	41.5	38.4	13.0	16.6	28.5	21.8	48.8	56.3	14.9	10.9	4.6	5.2
B-3-50	n.a.	29.1	n.a.	16.6	n.a.	12.5	n.a.	56.3	n.a.	10.9	n.a.	5.2
B-3-40	n.a.	33.8	n.a.	14.5	n.a.	19.2	n.a.	71.8	n.a.	10.9	n.a.	5.2
B-3-30	n.a.	38.2	n.a.	12.5	n.a.	25.6	n.a.	45.4	n.a.	10.3	n.a.	5.2
B-3-20	n.a.	44.9	n.a.	12.4	n.a.	32.5	n.a.	81.6	n.a.	10.9	n.a.	5.2
B-3-10	n.a.	49.8	n.a.	9.6	n.a.	40.2	n.a.	71.2	n.a.	10.3	n.a.	5.2
B-3-0	n.a.	54.1	n.a.	8.8	n.a.	36.6	n.a.	53.4	n.a.	10.9	n.a.	4.6

Note: EXP = experimental; FEA = finite-element analysis; n.a. = not applicable. 1 in. = 25.4 mm; 1 lb = 4.448 N; 1 kip = 4.448 kN.

pared with the one-tendon beam, B-1, because of the high reinforcement ratio. For the strengthened beams, the elastic energy portion increased when the level of prestress in the CFRP laminates increased (Fig. 9). This is attributed to the fact that the prestressed CFRP laminates effectively reduced the stress levels in the internal prestressing strands, resulting in improved yield load capacities of the strengthened beams. It should be noted that the plastic energy itself in the strengthened beams increased due to the contribution of the prestressed CFRP laminates (Table 4). The level of prestress in the CFRP laminates did not significantly affect the stiffness of the strengthened beams; however, the stiffness of the strengthened beams was considerably higher than that of the unstrengthened beam B-2 after yielding of the prestressing strands (Table 4).

Considering a ductility perspective, more plastic energy should be developed to increase ductility of the strengthened beams. Based on the observation in Fig. 9, the applied prestress level should be lower than about 30% to provide better ductility with respect to the unstrengthened beams. The practical significance of this observation is that an existing prestressed concrete beam may have better ductility when it is strengthened with prestressed CFRP laminates with a prestress level of up to 30%. More study on this issue may be necessary to draw a general conclusion by conducting more analyses using various sizes and reinforcement ratios of prestressed concrete beams.

Elastic recovery and residual deflections

The strengthened beams typically return to their initial state when the applied load is removed because of the elastic energy; however, permanent deformations of the beams may exist if the beams have been loaded beyond their yield capacities. This elastic recovery may indirectly represent the level of damage. For example, a large amount of the elastic recovery means that the beam is relatively less damaged under the applied load.

For this study, the elastic recovery was assumed as the area under the load-steel strain curve and the corresponding ratio was defined as the recovery area to the total area (**Fig. 10**). The unloading curve in Fig. 10 was obtained by subtracting the unrecoverable plastic strain from the total strain in the prestressing strands, given that a direct unloading curve was not available in the FEA. The strengthened beams with prestress levels higher than 20% showed lower elastic recovery ratios compared with the
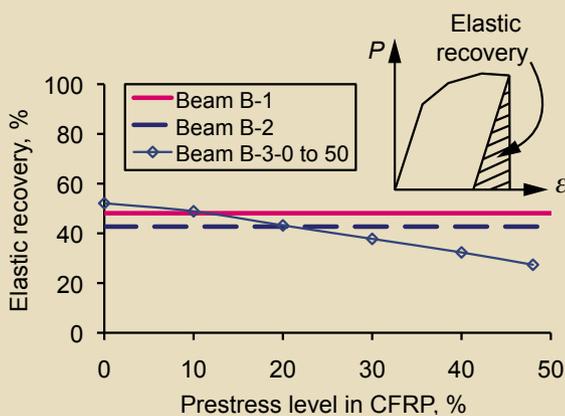

Figure 10. This graph shows the elastic recovery of the beams. Note: CFRP = carbon-fiber-reinforced polymer; P = applied load; ε = steel strain.
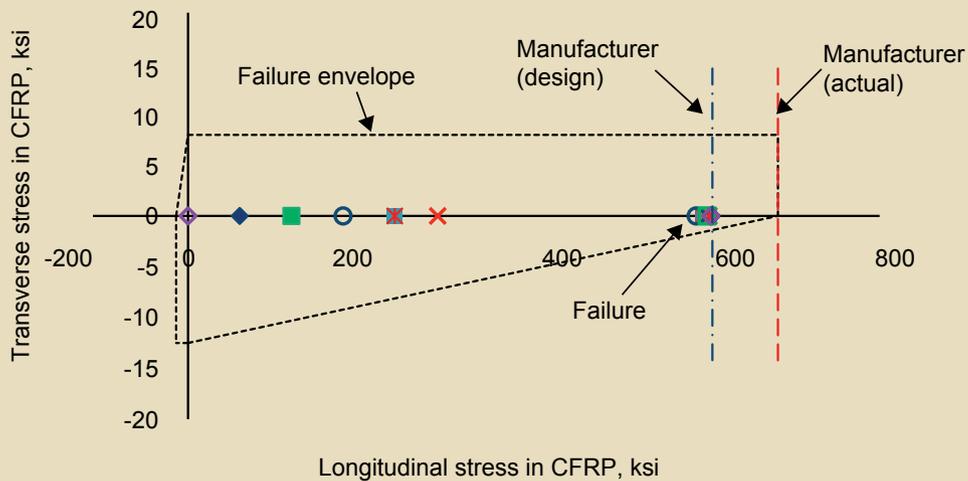
Figure 11. This graph illustrates the Mohr-Coulomb failure criterion for the carbon-fiber-reinforced-polymer laminates. Note: CFRP = carbon-fiber-reinforced polymer. 1 ksi = 6.895 MPa.

unstrengthened beams (Fig. 10). This observation implies that the prestressed concrete beams strengthened with a prestress level of higher than 20% experienced more damage compared with the unstrengthened beams, even though the ultimate loads of the strengthened beams were almost the same.

Failure criterion

Determination of an adequate failure condition of a CFRP-strengthened beam is an important issue for design professionals. The Mohr-Coulomb failure theory is particularly applicable to brittle materials, provided that the internal friction is a significant factor to the critical shear stress. For this study, a state of plane stress was assumed to represent the thin CFRP laminates, which had a thickness t of 0.0065 in. (0.165 mm). The Mohr-Coulomb failure theory assumes that the critical shear stress τ is a function of the normal stress σ on a shear plane:[17]

$$\tau = C_1\sigma + C_2$$

where

C_1 = material constant

C_2 = material constant

The constants may be solved by providing unidirectional failure strengths. The failure criterion for a plane stress condition is, therefore, expressed as follows:[17]

$$\frac{\sigma_1}{\sigma_{ut}} - \frac{\sigma_2}{\sigma_{ut}} = 1 \text{ for } \sigma_1 > 0 \text{ and } \sigma_2 < 0$$

$\sigma_1 = \sigma_{ut}$ and $\sigma_2 = \sigma_{ut}$ for $\sigma_1 > 0$ and $\sigma_2 > 0$

$\sigma_1 = \sigma_{uc}$ and $\sigma_2 = \sigma_{uc}$ for $\sigma_1 < 0$ and $\sigma_2 < 0$

where

σ_1 = stress in the longitudinal (fiber direction) of the CFRP laminate

σ_2 = stress in the transverse direction of the CFRP laminate

σ_{ut} = maximum tensile strength of the laminate

σ_{uc} = maximum compressive strength of the laminate

It was assumed that the carbon fiber could not resist compressive forces, so the compressive strength of the resin governed the compression failure of the CFRP laminate. The failure of the CFRP laminate occurred when the stress state was outside the failure envelope. For this study, fiber contributions, such as fiber kinking in σ_{uc} and the lateral deformation of the fibers, were ignored. The failure criterion is to be used as an indicator of failure, rather than an absolute predictor.[18]

Figure 11 shows the computed Mohr-Coulomb failure envelope for the CFRP laminates, including detailed stress values obtained from strengthened beams B-3-0 to B-3-50 and the failure strengths reported by the manufacturer. The stress interaction between σ_1 (fiber direction) and σ_2 (perpendicular to the fiber direction) was not explicitly taken into account in the stresses obtained from the FEA because the CFRP laminates were simplified to uniaxial elements for computational simplicity; however, the failure prediction by the Mohr-Coulomb failure theory was satisfactory.

Conclusion

This paper presents the effect of various prestressing levels on the prestressed CFRP laminates for strengthening prestressed concrete beams. Calibrated nonlinear 3-D FEA models were used to predict the flexural behavior

of the strengthened beams, including the load-deflection responses, strain development of the reinforcement, elastic and plastic behavior, and failure predictions. The proposed strengthening technique may be recommended for post-tensioning existing concrete structures. Several conclusions were made from this study:

- The prestress levels in the prestressed CFRP laminates significantly influence load-deflection responses of the strengthened prestressed concrete beams. The midspan deflection at failure was particularly affected by the prestress levels despite the same ultimate-load-carrying capacities of the beams, whereas the contribution of the prestress levels to the deflections at cracking and yield loads was not remarkable.

- The usable strains in the prestressed CFRP laminates are the governing factor in determining the plastic strain development in the internal prestressing strands, taking into account the stress-sharing mechanism between the reinforcement. The level of prestress in the CFRP laminates significantly affects plastic energy (ductility) and elastic recovery (damage) of the strengthened beams. A prestress level between 20% and 30% of the ultimate design strain of CFRP laminates is recommended for prestressed concrete beams strengthened with prestressed CFRP laminates. More research may be necessary to draw a general conclusion regarding this recommendation.

- The failure of strengthened beams is primarily due to the rupture of CFRP laminates. The Mohr-Coulomb failure criterion provided reasonable predictions for such a failure mode of strengthened beams, though the axial and lateral stress interaction of the CFRP laminates was ignored for computational convenience.

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Notation

A_f = cross-sectional area of carbon-fiber-reinforced-polymer laminate

A_p = cross-sectional area of prestressing strands

C_1 = material constant

C_2 = material constant

$E_{elastic}$ = energy absorbed until a beam yields

$E_{plastic}$ = energy absorbed after yielding of a beam up to its failure

E_{total} = total energy absorbed until a beam fails

f = stress

f_{fu} = ultimate strength of carbon-fiber-reinforced polymer

f_{pu} = ultimate strength of prestressing strands

l = length of carbon-fiber-reinforced-polymer laminate

L = undeformed element length or beam length

P = applied load

P_{cr} = cracking load

$P_j(x)$ = jacking force

P_{ult} = ultimate load

P_{yld} = yield load

t = thickness

δ_{cr} = deflection at cracking load

δ_{ult} = deflection at ultimate load

δ_{yld} = deflection at yield load

Δ = difference between the prestrained element length and the zero-strain length

ε = steel strain

ε_{fu} = ultimate failure strain of carbon-fiber-reinforced-polymer laminate

ε_{pu} = ultimate strain of prestressing steel

σ = normal stress

σ_1 = stress in the longitudinal (fiber) direction of the carbon-fiber-reinforced-polymer laminate

σ_2 = stress in the transverse direction of the carbon-fiber-reinforced-polymer laminate

σ_{uc} = maximum compressive strength of the laminate

σ_{ut} = maximum tensile strength of the laminate

τ = shear stress

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Synopsis

This paper presents the effect of prestress levels in prestressed, carbon-fiber-reinforced-polymer (CFRP) laminates for strengthening prestressed concrete beams based on a validated, nonlinear, three-dimensional finite-element-analysis model.

A total of nine medium-scale prestressed concrete beams, six of which were externally strengthened with prestressed CFRP laminates, were studied to examine the contribution of various prestressing levels in the CFRP laminates to flexural behavior of the strengthened beams.

The focus of this study was on the load-deflection responses, strain development in the internal and external reinforcement, elastic and plastic energy characteristics, and failure predictions. For a practical use of this type of strengthening method, the recommended level of prestress is between 20% and 30% of the ultimate design strain of the CFRP laminates.

More research, however, may be warranted to draw a general design guideline for the prestress level, including various size effects and reinforcement ratios of prestressed concrete beams.

Keywords

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

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